Class XI- MATHEMATICS
Chapter-2 : RELATIONS and FUNCTIONS
Hand out of Module 1/2

Learning Outcome:

In this module we are going to learn about

- Cartesian Products of Sets.
- Relations.
- Number of Relations from a set A to a set B.
- Domain, Co-domain and Range of a Relation.

Introduction:

Let us consider the following problem.

Mohan has 2 pants P_1 and P_2 and 3 shirts S_1, S_2 and S_3. How many different pairs of a pant and a shirt, can he dress up with?

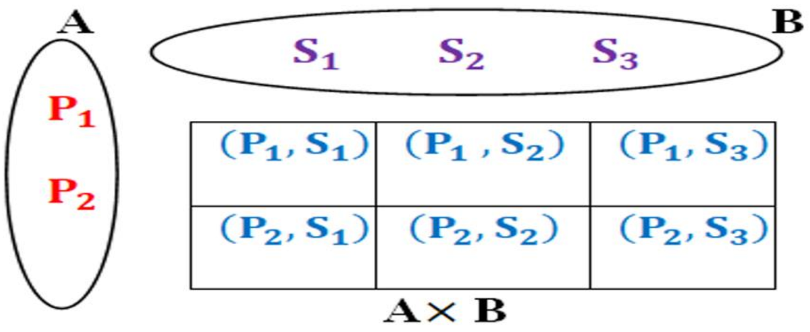
Let A be the set of pants and B be the set of shirts.

Then A ={P_1, P_2} and B = { S_1, S_2}.

We can see that Mohan can make 6 distinct pairs as given below:

$(P_1, S_1), (P_1, S_2), (P_1, S_3), (P_2, S_1), (P_2, S_2), (P_2, S_3)$

This can be represented by the following diagram



Cartesian Products of Sets :

Given two non-empty sets A and B. The cartesian product A × B is the set of all ordered pairs of elements from A and B, i.e., $A \times B = \{ (a, b) : a \in A,\ b \in B \}$

Note :

(i) . If either A or B is an empty set, then A × B will also be an empty set, i.e., $A \times B = \emptyset$

(ii). Two ordered pairs are equal, if and only if the corresponding first elements are equal and the second elements are also equal.

(iii). If there are p elements in A and q elements in B, then there will be pq elements in $A \times B$,

i.e., if $n(A) = p$ and $n(B) = q$, then $n(A \times B) = pq$.

(iv). If A and B are non-empty sets and either A or B is an infinite set, then so is $A \times B$.

(v). $A \times A \times A = \{(a, b, c) : a, b, c \in A\}$. Here (a, b, c) is called an ordered triplet.

(vi). In general, $A \times B \neq B \times A$

(vii**). $A \times (B \cap C) = (A \times B) \cap (A \times C)$ and $A \times (B \cup C) = (A \times B) \cup (A \times C)$**

Example 1:

Let A= {1, 2} and B ={ a, b, c}, find $A \times B$.

Solution: $A \times B = \{(1, a), (1, b), (1, c), (2, a), (2, b), (2, c)\}$

Example 2:

If, $(a - 3, b + 2) = (4, -2)$, find the values of a and b.

Solution : Since the ordered pairs are equal, the corresponding elements are equal.

Therefore $a - 3 = 4$ and $b + 2 = -2$. Solving we get $a = 7$ and $b = -4$.

Example 3:

If, P = {a, b} and Q = {x, y}, find $P \times Q$ and $Q \times P$. Are these two products equal?

Solution: $P \times Q = \{(a, x), (a, y), (b, x), (b, y)\}$ and $Q \times P = \{(x, a), (x, b), (y, a), (y, b)\}$

Since, by the definition of equality of ordered pairs, the pair (a, x) is not equal to the pair (x, a), we conclude that $P \times Q \neq Q \times P$.

Example 4:

Let A = {1, 2}, B = {3, 4}, C = {4, 5} Verify that $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

Solution: $B \cap C = \{4\}$. Therefore, $A \times (B \cap C) = \{1, 2\} \times \{4\} = \{(1,4), (2,4)\}$……………….(1)

Also, $A \times B = \{(1, 3),(1,4), (2,3),(2,4)\}$, $A \times C = \{(1, 4),(1,5), (2,4), (2,5)\}$………………..…(2)

Therefore, $(A \times B) \cap (A \times C) = \{(1,4), (2,4)\}$

Hence, from (1) & (2), we get, $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

Example 5:

If $A \times B = \{(x, q), (x, r), (y, q), (y, r)\}$, find A and B.

Solution: A = set of first elements = {x, y}

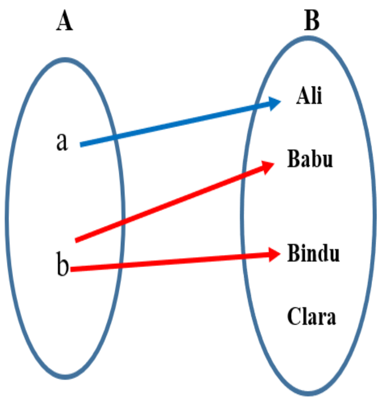
B = set of second elements = {q, r}.

<u>Note</u>:

(i) The Cartesian product $R \times R = \{(x, y) : x, y \in R\}$ represents the coordinates of all the points in two-dimensional space.

(ii)The cartesian product $R \times R \times R = \{(x, y, z) : x, y, z \in R\}$ represents the coordinates of all the points in three-dimensional space.

RELATIONS



Introduction:

Consider the sets A = {a, b} and B = {Ali, Babu, Bindu, Clara}

A × B = {(a, Ali), (a, Babu), (a, Bindu), (a, Clara), (b, Ali), (b, Babu), (b, Bindu), (b, Clara)}.

Consider a subset 'R' of A × B by introducing a relation between the first element x and the second element y of each ordered pair (x, y) as

R= { (x, y): x is the first letter of the name y, x ∈ A, y ∈ B}.

Then R = {(a, Ali), (b, Babu), (b, Bindu)}

A visual representation of this relation R is shown in the adjacent diagram.

Relation: A relation R from a non-empty set A to a non-empty set B is a subset of the cartesian product A × B.

The subset is derived by describing a relationship between the first element and the second element of the ordered pairs in A × B. The second element is called the *image* of the first element.

Domain of a relation: The set of all first elements of the ordered pairs in a relation R from a set A to a set B is called the domain of the relation R.

Range of a relation: The set of all second elements in a relation R from a set A to a set B is called the range of the relation R.

Codomain of a relation: The whole set B is called the codomain of the relation R.

Note

(i). range ⊂ codomain.

(ii). A relation may be represented algebraically either by Roster method or by Set- builder method.

(iii). An arrow diagram is a visual representation of a relation.

(iv). If n(A) = p and n(B) = q, then n (A × B) = pq.

Therefore, the total number of relations from A to B is 2^{pq}.

(v). A relation R from A to A is also stated as a relation on A.

Example 1:

Let A = {1, 2, 3, 4, 5}. Define a relation R from A to A by R = {(x , y) : y = x + 2 }

(i). Write the relation in roster form.

(ii). Write down the domain, codomain and range of R. Is range a subset of co domain?

(iii). Depict this relation using an arrow diagram.

Solution:

(i). The relation, $R = \{(1, 3), (2, 4), (3, 5)\}$.

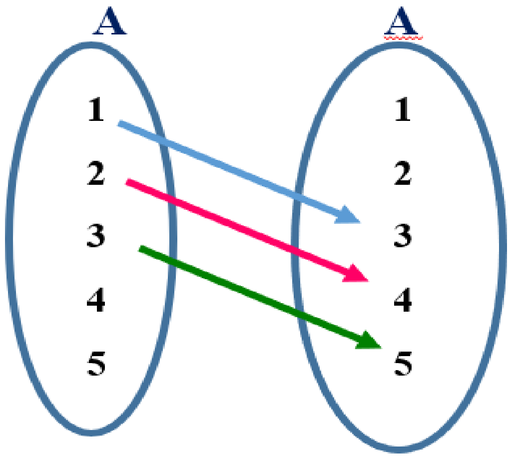
(ii). domain of $R = \{1, 2, 3\}$

codomain of $R = \{1, 2, 3, 4, 5\}$

range of $R = \{3, 4, 5\}$

Yes, Range is a subset of co- domain

(iii). The corresponding arrow diagram is shown in the adjacent figure.



Example 2:

Let $A = \{1, 2\}$ and $B = \{a, b, c\}$. Find the number of relations from A to B.

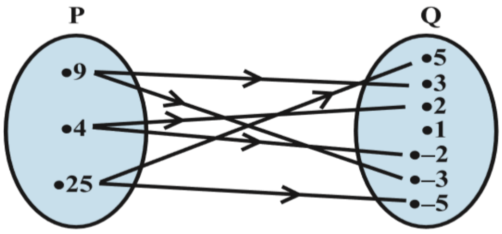
Solution: We have, $A \times B = \{(1, a), (1, b), (1, c), (2, a), (2, b), (2, c)\}$.

Since $n(A\times B) = 6$. Therefore, the number of relations from A to B will be $2^6 = 64$.

Example:

The adjacent figure shows a relation between the sets P and Q. Write this relation (i) in set-builder form, (ii) in roster form.

(iii) What is its domain and range?



Solution: (i). In set-builder form, $R = \{(x , y): x \text{ is the square of } y, x \in P, y \in Q\}$

(ii). In roster form, $R = \{(9, 3), (9, –3), (4, 2), (4, –2), (25, 5), (25, –5)\}$

(iii). The domain $= \{4, 9, 25\}$. The range $= \{– 2, 2, –3, 3, –5, 5\}$.

What have we learned?

- **Ordered pair**: A pair of elements grouped together in a particular order.
- **Cartesian product**: Cartesian product of two sets A and B is given by $A \times B = \{(a, b): a \in A, b \in B\}$
- In particular $R \times R = \{(x, y): x, y \in R\}$ and $R \times R \times R = \{(x, y, z): x, y, z \in R\}$
- If $(a, b) = (x, y)$, then $a = x$ and $b = y$.
- If $n(A) = p$ and $n(B) = q$, then $n(A \times B) = pq$.
- $A \times \phi = \phi$
- In general, $A \times B \neq B \times A$.
- **Relation**: A relation R from a set A to a set B is a subset of the cartesian product $A \times B$.
- **Domain**: The domain of R is the set of all first elements of the ordered pairs in a relation R.
- **Range**: The range of the relation R is the set of all second elements of the ordered pairs in a relation R.

****